

Números Rep 1

1)

$$A = (27^4)^9 - (25^3)^6 = 37$$

Agrupamos los exponentes:

$$A = 27^{36} - 25^{18} = 27^{36} - 5^{36}$$

Trabajamos en $\mathbb{Z}/37$ (Para simplificar la notación) a) $27^{36} \bmod 37$

$$27^0 \bmod 37 = 1$$

$$27^1 \bmod 37 = 27$$

$$27^2 \bmod 37 = 729 = 26$$

$$27^3 \bmod 37 = 27 \cdot 26 = 702 = 36$$

$$27^4 \bmod 37 = 27 \cdot 36 = 972 = 10$$

$$27^5 \bmod 37 = 27 \cdot 10 = 270 = 11$$

$$27^6 \bmod 37 = 27 \cdot 11 = 297 = 1$$

Luego se cumple

$$27^n \bmod 37 = \begin{cases} 1 & \text{si } n = 0 \bmod 6 \\ 27 & \text{si } n = 1 \bmod 6 \\ 26 & \text{si } n = 2 \bmod 6 \\ 36 & \text{si } n = 3 \bmod 6 \\ 10 & \text{si } n = 4 \bmod 6 \\ 11 & \text{si } n = 5 \bmod 6 \end{cases}$$

$$27^{36} \Rightarrow \boxed{27^{36} \bmod 37 = 1} \text{ pues}$$

se cumple que $36 = 0 \bmod 6$

$$b) 5^{36} \bmod 37$$

$$5^0 \bmod 37 = 1$$

$$5^1 \bmod 37 = 5$$

$$5^2 \bmod 37 = 25$$

$$5^3 \bmod 37 = 125 = 14$$

$$5^4 \bmod 37 = 5 \cdot 14 = 70 = 33$$

$$5^5 \bmod 37 = 5 \cdot 33 = 165 = 17$$

$$5^6 \bmod 37 = 5 \cdot 17 = 85 = 11$$

$$5^7 \bmod 37 = 5 \cdot 11 = 55 = 18$$

$$5^8 \bmod 37 = 5 \cdot 18 = 90 = 16$$

$$5^9 \bmod 37 = 5 \cdot 16 = 80 = 6$$

$$5^{10} \bmod 37 = 5 \cdot 6 = 30 = 30$$

$$5^{11} \bmod 37 = 5 \cdot 30 = 150 = 2$$

$$5^{12} \bmod 37 = 5 \cdot 2 = 10 = 10$$

$$5^{13} \bmod 37 = 5 \cdot 10 = 50 = 13$$

$$5^{14} \bmod 37 = 5 \cdot 13 = 65 = 28$$

$$5^{15} \bmod 37 = 5 \cdot 28 = 140 = 29$$

$$5^{16} \bmod 37 = 5 \cdot 29 = 145 = 34$$

$$5^{17} \bmod 37 = 5 \cdot 34 = 170 = 22$$

$$5^{18} \bmod 37 = 5 \cdot 22 = 110 = 36$$

$$5^{19} \bmod 37 = 5 \cdot 36 = 180 = 32$$

$$5^{20} \bmod 37 = 5 \cdot 32 = 160 = 12$$

$$5^{21} \bmod 37 = 5 \cdot 12 = 60 = 23$$

$$5^{22} \bmod 37 = 5 \cdot 23 = 115 = 4$$

$$5^{23} \bmod 37 = 5 \cdot 4 = 20 = 20$$

$$5^{24} \bmod 37 = 5 \cdot 20 = 100 = 26$$

$$5^{25} \bmod 37 = 5 \cdot 26 = 130 = 19$$

$$5^{26} \bmod 37 = 5 \cdot 19 = 95 = 21$$

$$5^{27} \bmod 37 = 5 \cdot 21 = 105 = 31$$

$$5^{28} \bmod 37 = 5 \cdot 31 = 155 = 7$$

$$5^{29} \bmod 37 = 5 \cdot 7 = 35$$

$$5^{30} \bmod 37 = 5 \cdot 35 = 27$$

$$5^{31} \bmod 37 = 5 \cdot 27 = 135 = 24$$

$$5^{32} \bmod 37 = 5 \cdot 24 = 120 = 9$$

$$5^{33} \bmod 37 = 5 \cdot 9 = 45 = 8$$

$$5^{34} \bmod 37 = 5 \cdot 8 = 40 = 3$$

$$5^{35} \bmod 37 = 5 \cdot 3 = 15$$

$$5^{36} \bmod 37 = 5 \cdot 15 = 75 = 1$$

Por tanto

$$A \bmod 37 = (27^{36} - 5^{36}) \bmod 37 =$$

$$= 27^{36} \bmod 37 - 5^{36} \bmod 37 = 1 - 1 = 0$$

Luego $A = 27$



Más sencillo:

El pequeño teorema de Fermat

Asegura que dado un $n \in \mathbb{N}$

y una potencia $p \in \mathbb{N}$ primo y

no divisor de a se cumple

$$a^{p-1} = 1 \bmod p \rightarrow$$

$$27^{37-1} = 1 \bmod 37$$

$$5^{37-1} = 1 \bmod 37$$

$$A \bmod 37 = 1 - 1 = 0 \bmod 37$$