

① a) $m(x) = x^2 + x - 2 = (x+2)(x-1)$ B

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot (-2)}}{2}; \quad x = \frac{-1 \pm \sqrt{9}}{2}; \quad x = \frac{-1 \pm 3}{2} \begin{matrix} \nearrow -2 \\ \searrow 1 \end{matrix}$$

Raíz	Factor
-2	$x+2$
1	$x-1$

$$M(x) = x^5 + 2x^4 - x - 2 = (x-1)(x+1)(x+2)(x^2+1)$$

Raíz	Factor
1	$x-1$
-1	$x+1$
-2	$x+2$

$$\begin{array}{r|rrrrrr} & 1 & 2 & 0 & 0 & -1 & -2 \\ 1 & & 1 & 3 & 3 & 3 & 2 \\ \hline & 1 & 3 & 3 & 3 & 2 & 0 \end{array}$$

$$x^4 + 3x^3 + 3x^2 + 3x + 2$$

$$\begin{array}{r|rrrrr} & 1 & 3 & 3 & 3 & 2 \\ -1 & & -1 & -2 & -1 & -2 \\ \hline & 1 & 2 & 1 & 2 & 0 \end{array}$$

$$x^3 + 2x^2 + x + 2$$

$$\begin{array}{r|rrrr} & 1 & 2 & 1 & 2 \\ -2 & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$x^2 + 1$$

$$x = \pm \sqrt{-1} \notin \mathbb{R}$$

b) $P(x)?$

$Q(x)?$

$\left\{ \begin{array}{l} \text{M.C.D} = x^2 + x - 2 = (x+2)(x-1) \\ \text{m.c.m} = x^5 + 2x^4 - x - 2 = (x-1)(x+1)(x+2)(x^2+1) \end{array} \right.$

• Ambos polinomios tienen como factores $(x+2)(x-1)$

$\left\{ \begin{array}{l} P(x) = (x+2)(x-1)(x+1) \\ Q(x) = (x+2)(x-1)(x^2+1) \end{array} \right.$

$\left\{ \begin{array}{l} P(x) = (x+2)(x-1)(x+1)(x^2+1) \\ Q(x) = (x+2)(x-1) \end{array} \right.$

B

② a) $\frac{x^2-3}{x^4-2x^3+x^2}$

grado numerador < grado denominador

* factorizar denominador

$$x^4-2x^3+x^2 = x^2(x^2-2x+1)$$

$$x = \frac{2 \pm \sqrt{4-4 \cdot 1}}{2}; x = \frac{2 \pm 0}{2} \rightarrow \begin{matrix} 1 \\ 1 \end{matrix}$$

Raíz	Factor
0	x (doble)
1	x-1 (doble)

$$\hookrightarrow \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$= \frac{x^2-3}{x^4-2x^3+x^2} = \frac{Ax(x-1)^2 + B(x-1)^2 + C(x-1)x^2 + Dx^2}{x^2(x-1)^2}$$

$$x^2-3 = Ax(x-1)^2 + B(x-1)^2 + C(x-1)x^2 + Dx^2$$

$$x=1 \rightarrow \boxed{-2 = D}$$

$$x=0 \rightarrow \boxed{-3 = B}$$

$$x=-1 \rightarrow -2 = -4A + 4B - 2C + D$$

$$-2 = -4A + 4(-3) - 2C - 2$$

$$-2 = -4A - 12 - 2C - 2$$

$$-4A - 12 - 2C = 0$$

$$x=2 \rightarrow 1 = 2A + B + 4C + 4D$$

$$1 = 2A - 3 + 4C + 4(-2)$$

$$1 = 2A - 3 + 4C - 8$$

$$12 = 2A + 4C$$

SISTEMA

$$-4A - 2C = 12$$

$$2A + 4C = 12$$

} $\rightarrow$ Reducción

$$-4A - 2C = 12$$

$$(\cdot 2) + 4A + 8C = 24$$

$$6C = 36$$

$$\boxed{C = 6}$$

$$2A + 4 \cdot 6 = 12$$

$$2A = -12$$

$$\boxed{A = -6}$$

$$\frac{-6}{x} - \frac{3}{x^2} + \frac{6}{x-1} - \frac{2}{(x-1)^2}$$

an

$$b) \frac{x^3 + 2x}{x^3 + 2x^2 + 6x + 5} = 1 + \frac{-2x^2 - 4x - 5}{x^3 + 2x^2 + 6x + 5}$$

grado numerador = grado denominador

* división

$$\begin{array}{r} x^3 \quad 2x \\ -x^3 - 2x^2 - 6x - 5 \\ \hline -2x^2 - 4x - 5 \end{array} \quad \begin{array}{r} x^3 + 2x^2 + 6x + 5 \\ 1 \end{array}$$

* factorizar denominador

$$x^3 + 2x^2 + 6x + 5 = (x+1)(x^2 + x + 5)$$

$$\begin{array}{r|rrrr} & 1 & 2 & 6 & 5 \\ -1 & & -1 & -1 & -5 \\ \hline & 1 & 1 & 5 & 0 \end{array}$$

$$x^2 + x + 5$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 5}}{2} = \frac{-1 \pm \sqrt{-19}}{2} \notin \mathbb{R}$$

$$\frac{A}{x+1} + \frac{Bx+C}{x^2+x+5}$$

$$+ \frac{-2x^2 - 4x - 5}{x^3 + 2x^2 + 6x + 5} = 1 + \frac{A(x^2+x+5) + (Bx+C)(x+1)}{(x+1)(x^2+x+5)}$$

$$-2x^2 - 4x - 5 = A(x^2+x+5) + (Bx+C)(x+1)$$

$$x = -1 \rightarrow -3 = 5A ; \boxed{A = -\frac{3}{5}} \quad B$$

$$x = 0 \rightarrow -5 = 5A + C$$

$$-5 = \cancel{5} \cdot \left(-\frac{3}{\cancel{5}}\right) + C$$

$$x = 1 \rightarrow -11 = 7A + 2(B+C) \quad -5 + 3 = C ; \boxed{C = -2} \quad B$$

$$-11 = 7\left(-\frac{3}{5}\right) + 2B + 2(-2)$$

$$-11 = -\frac{21}{5} - 4 + 2B$$

$$-\frac{14}{5} = 2B ;$$

$$\boxed{B = -\frac{7}{5}} \quad B$$

$$1 + \frac{-3/5}{x+1} + \frac{-7/5x - 2}{x^2+x+5}$$

③ $P(x)$ grado 4

2 raíces enteras simples

Término independiente (-1)

Múltiplo de x^2+1

$$(ax^4 + bx^3 + cx^2 + d(x) - 1)$$

$$(x-i)(x-h)(x^2+1)$$

$$(x-1)(x+1)(x^2+1) =$$

$$= (x^2-1)(x^2+1)$$

$$= \boxed{x^4 - 1}$$

$$\boxed{P(x) = x^4 - 1}$$

$$ax^4 + bx^3 + cx^2 + dx + e$$

la multiplicación de
 $-i \cdot (-h) \cdot 1 = \text{término ind} = -1$

$$-i \cdot (-h) = -1$$

$$\begin{cases} i = 1 \\ h = -1 \end{cases}$$

④ $P(x) = mx^3 - x^2 + 3mx - 5$

$x+1$

$m?$

$R = 2$

Teorema del resto

$$\begin{matrix} x+1 \\ x=-1 \end{matrix}$$

$$P(-1) = m(-1)^3 - (-1)^2 + 3m(-1) - 5 = 2$$

$$-m - 1 - 3m - 5 = 2$$

$$-4m - 6 = 2$$

$$-4m = 8$$

$$\boxed{m = -2}$$

División Ruffini

$$\begin{array}{r|rrrr} & -2 & -1 & -6 & -5 \\ -1 & & 2 & -1 & 7 \\ \hline & -2 & 1 & -7 & 2 \end{array}$$

$$C(x) = -2x^2 + x - 7$$

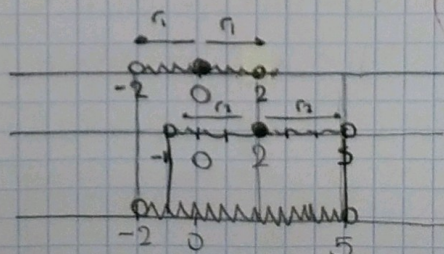
$R = 2$

⑤ a) $r_1, r_2?$

$$E_1(0, r_1)$$

$$E_2(2, r_2)$$

$$E_1 \cup E_2 = (-2, 5)$$



$$R_1 = 2 \quad E_1(0, 2)$$

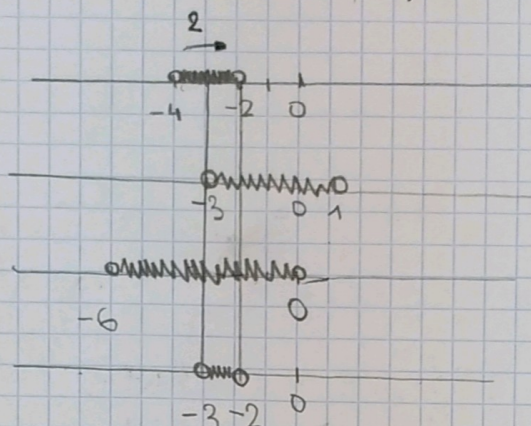
$$R_2 = 3 \quad E_2(2, 3)$$

$$b) E^+(-4, 2) \cap (-3, 1) \cap (-6, 0) = (-3, -2)$$

$$E^+(-4, 2)$$

$$(-3, 1)$$

$$(-6, 0)$$



$$\begin{aligned} \textcircled{6} \quad a) \quad \frac{(\sqrt[4]{8} : 2^{-3}) \cdot 0,5^{-2}}{16^{-1/5} : \sqrt{32}} &= \frac{(\sqrt[4]{2^3} : 2^{-3}) \cdot (\frac{1}{2})^{-2}}{(2^4)^{-1/5} : \sqrt{2^5}} = \\ &= \frac{(2^{3/4} : 2^{-3}) \cdot 2^2}{2^{-4/5} : 2^{5/2}} = \frac{2^{3/4+3+2}}{2^{-4/5-5/2}} = \frac{2^{\frac{3+12+8}{4}}}{2^{\frac{-9-25}{10}}} \end{aligned}$$

$$= \frac{2^{\frac{23}{4}}}{2^{-\frac{33}{10}}} = 2^{\frac{23}{4} + \frac{33}{10}} = 2^{\frac{115+66}{20}} = 2^{\frac{181}{20}} = \sqrt[20]{2^{181}} = 2^9 \sqrt[20]{2}$$

$$\begin{aligned} b) \quad (\sqrt{14+\sqrt{7}-\sqrt{81}})^{-1/2} &= (\sqrt{14+\sqrt{7}-3})^{-1/2} = \\ &= (\sqrt{14+2})^{-1/2} = (4)^{-1/2} = \left(\frac{1}{4}\right)^{1/2} = \sqrt{\frac{1}{4}} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} c) \quad 3\sqrt[3]{16} - \frac{3}{4}\sqrt[3]{128} - \sqrt[3]{24} - \sqrt[9]{8} + 5\sqrt[3]{81} &= \\ &= 3\sqrt[3]{2^4} - \frac{3}{4}\sqrt[3]{2^7} - \sqrt[3]{2^3 \cdot 3} - \sqrt[9]{2^3} + 5\sqrt[3]{3^4} = \\ &= 6\sqrt[3]{2} - \frac{3}{4} \cdot 2\sqrt[3]{2} - 2\sqrt[3]{3} - \sqrt[3]{2} + 15\sqrt[3]{3} \\ &= (6-3-1)\sqrt[3]{2} + (-2+15)\sqrt[3]{3} = \\ &= 2\sqrt[3]{2} + 13\sqrt[3]{3} \end{aligned}$$

$$d) (2\sqrt{50} - 3\sqrt{2})^2 - (\sqrt{50} - 4\sqrt{2})(\sqrt{50} + 4\sqrt{2})$$

$$= [(2\sqrt{50})^2 + (3\sqrt{2})^2 - (2 \cdot 2 \cdot 3 \cdot \sqrt{50} \cdot \sqrt{2})] - [(\sqrt{50})^2 - (4\sqrt{2})^2] =$$

$$= (4 \cdot 50 + 9 \cdot 2 - 12\sqrt{100}) - (50 - 16 \cdot 2) =$$

$$= (200 + 18 - 12 \cdot 10) - (50 - 32)$$

$$98 - 18 = \boxed{80} \text{ B}$$

$$\textcircled{7} \text{ a) } \frac{5\sqrt{8} - 3\sqrt{2}}{\sqrt{18}} = \frac{5\sqrt{8} - 3\sqrt{2}}{\sqrt{3^2 \cdot 2}} = \frac{5\sqrt{8} - 3\sqrt{2}}{3\sqrt{2}} =$$

$$= \frac{(5\sqrt{8} - 3\sqrt{2}) \sqrt{2}}{3\sqrt{2} \cdot \sqrt{2}} = \frac{5\sqrt{16} - 3(\sqrt{2})^2}{3 \cdot 2} = \frac{5 \cdot 4 - 3 \cdot 2}{6} =$$

$$= \frac{20 - 6}{6} = \frac{14}{6} = \boxed{\frac{7}{3}} \text{ B}$$

$$\text{b) } \frac{\sqrt{2} + 2\sqrt{3}}{\sqrt{2} - 2\sqrt{3}} = \frac{(\sqrt{2} + 2\sqrt{3})(\sqrt{2} + 2\sqrt{3})}{(\sqrt{2} - 2\sqrt{3})(\sqrt{2} + 2\sqrt{3})} =$$

$$= \frac{(\sqrt{2} + 2\sqrt{3})^2}{(\sqrt{2})^2 - (2\sqrt{3})^2} = \frac{(\sqrt{2})^2 + (2\sqrt{3})^2 + 2 \cdot 2\sqrt{2} \cdot \sqrt{3}}{2 - 4 \cdot 3} =$$

$$= \frac{2 + 12 + 4\sqrt{6}}{-10} = \frac{14 + 4\sqrt{6}}{-10} = \boxed{\frac{7 + 2\sqrt{6}}{-5}} \text{ B}$$